# J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

TOPIC : CONTINUITY + REGRESSION — SYJC TEST - 02 - SET 2 — DURATION - 1 $^1/_2$ HR — MARKS - 40

## SOLUTION SET
## SECTION - I

Q-1

**01.** $f(x) = \dfrac{\sqrt{4+x}-2}{3x}$ ; $x \neq 0$

$= 1/4$ ; $x = 0$

Discuss continuity at x = 0

**SOLUTION :**

**STEP 1**

$\lim_{x \to 0} f(x)$

$= \lim_{x \to 0} \dfrac{\sqrt{4+x}-2}{3x}$

$= \lim_{x \to 0} \dfrac{\sqrt{4+x}-2}{3x} \cdot \dfrac{\sqrt{4+x}+2}{\sqrt{4+x}+2}$

$= \lim_{x \to 0} \dfrac{4+x-4}{3x} \cdot \dfrac{1}{\sqrt{4+x}+2}$

$= \lim_{x \to 0} \dfrac{\cancel{x}}{3\cancel{x}} \cdot \dfrac{1}{\sqrt{4+x}+2}$ $x \neq 0$

$= \lim_{x \to 0} \dfrac{1}{3} \cdot \dfrac{1}{\sqrt{4+x}+2}$

$= \dfrac{1}{3} \cdot \dfrac{1}{\sqrt{4+0}+2}$

$= \dfrac{1}{3} \cdot \dfrac{1}{2+2}$

$= \dfrac{1}{12}$

**STEP 2 :**

$f(0) = 1/4$ ……… given

**STEP 3 :**

$f(0) \neq \lim_{x \to 0} f(x)$ ; *f* is discontinuous at x = 0

**STEP 4 :**

**REMOVABLE DISCONTINUITY**

*f* can be made continuous at x = 0 by redefining it as

$f(x) = \dfrac{\sqrt{4+x}-2}{3x}$ ; $x \neq 0$

$= 1/12$ ; $x = 0$

**02.** $f(x) = \dfrac{\log(1+3x)}{5x}$ ; $x \neq 0$

$= k$ ; $x = 0$

find k if the f is continuous at x = 0

**SOLUTION :**

**STEP 1**

$\lim_{x \to 0} f(x)$

$= \lim_{x \to 0} \dfrac{\log(1+3x)}{5x}$

$= \lim_{x \to 0} \dfrac{3}{5} \cdot \dfrac{\log(1+3x)}{3x}$

$= \dfrac{3(1)}{5}$

$= 3/5$

**STEP 2 :**

$f(0) = k$ ……… given

**STEP 3 :**

Since the f is continuous at x = 0 \

$f(0) = \lim_{x \to 0} f(x)$

$k = 3/5$

**03.** $f(x) = \dfrac{a^{2x} - 1}{x}$ ; $x \neq 0$

$= 2\log a$ ; $x = 0$

Discuss continuity at $x = 0$

**SOLUTION :**

**STEP 1**

$\lim\limits_{x \to 0} f(x)$

$= \lim\limits_{x \to 0} \dfrac{a^{2x} - 1}{x}$

$= \lim\limits_{x \to 0} 2\, \dfrac{a^{2x} - 1}{2x}$

$= 2.\log a$

**STEP 2 :**

$f(0) = 2.\log a$ ……… given

**STEP 3 :**

$f(0) = \lim\limits_{x \to 0} f(x)$

$\therefore f$ is continuous at $x = 0$

**04.** $f(x) = \dfrac{x\cos x + 3\tan x}{x^2 + \sin x}$ ; $x \neq 0$

$= 6$ ; $x = 0$

Discuss continuity at $x = 0$

**SOLUTION :**

**STEP 1**

$\lim\limits_{x \to 0} f(x)$

$= \lim\limits_{x \to 0} \dfrac{x\cos x + 3\tan x}{x^2 + \sin x}$

Divide Numerator & Denominator by $x$ , $x \neq 0$

$= \lim\limits_{x \to 0} \dfrac{\dfrac{x\cos x + 3\tan x}{x}}{\dfrac{x^2 + \sin x}{x}}$

$= \lim\limits_{x \to 0} \dfrac{\dfrac{x\cos x}{x} + 3\dfrac{\tan x}{x}}{\dfrac{x^2}{x} + \dfrac{\sin x}{x}}$

$= \lim\limits_{x \to 0} \dfrac{\cos x + 3\dfrac{\tan x}{x}}{x + \dfrac{\sin x}{x}}$

$= \dfrac{\cos 0 + 3(1)}{(0) + 1}$

$= 1 + 3$

$= 4$

**STEP 2 :**

$f(0) = 6$……… given

**STEP 3 :**

$f(0) \neq \lim\limits_{x \to 0} f(x)$

$\therefore f$ is discontinuous at $x = 0$

**STEP 4 :**

**REMOVABLE DISCONTINUITY**

$f$ can be made continuous at $x = 0$ by redefining it as

$f(x) = \dfrac{x\cos x + 3\tan x}{x^2 + \sin x}$ ; $x \neq 0$

$= 4$ ; $x = 0$

**Q2. Attempt any TWO of the following**

**(3 marks each) (6 marks)**

**01.** $f(x) = \dfrac{x^2 + 5}{x - 1}$ ; $1 < x \le 2$

$= kx + 1$ ; $x > 2$

if $f$ is continuous at $x = 2$ ; find k

**SOLUTION :**

**STEP 1**

$$\lim_{x \to 2-} f(x)$$

$$= \lim_{x \to 2} \frac{x^2 + 5}{x - 1}$$

$$= \frac{2^2 + 5}{2 - 1} = 9$$

**STEP 2**

$$\lim_{x \to 2+} f(x)$$

$$= \lim_{x \to 2} kx + 1$$

$$= k(2) + 1$$

$$= 2k + 1$$

**STEP 3**

$$f(2) = \frac{2^2 + 5}{2 - 1} = 9$$

**STEP 4**

Since $f$ is continuous at $x = 2$

$$\lim_{x \to 2-} f(x) = \lim_{x \to 2+} f(x) = f(2)$$

$$9 = 2k + 1 = 9$$

$$2k + 1 = 9$$

$$2k = 8$$

$$k = 4$$

**02.** $f(x) = \dfrac{10^x - 5^x - 2^x + 1}{x^2}$ ; $x \neq 0$

$= \log 10$ ; $x = 0$

Discuss continuity at $x = 0$

**SOLUTION :**

**STEP 1**

$$\lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} \frac{10^x - 5^x - 2^x + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{(5.2)^x - 5^x - 2^x + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{5^x.2^x - 5^x - 2^x + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{5^x(2^x - 1) - 1(2^x - 1)}{x^2}$$

$$= \lim_{x \to 0} \frac{(5^x - 1).(2^x - 1)}{x^2}$$

$$= \lim_{x \to 0} \frac{5^x - 1}{x} . \frac{2^x - 1}{x}$$

$$= \log 5 . \log 2$$

**STEP 2 :**

$f(0) = \log 10$ ……… given

**STEP 3 :**

$$f(0) \neq \lim_{x \to 0} f(x)$$

$\therefore$ $f$ is discontinuous at $x = 0$

**STEP 4 :**

**REMOVABLE DISCONTINUITY**

$f$ can be made continuous at $x = 0$ by redefining it as

$f(x) = \dfrac{10^x - 5^x - 2^x + 1}{x^2}$ ; $x \neq 0$

$= \log 5 . \log 2$ ; $x = 0$

**03.** $f(x) = \dfrac{\tan x - \sin x}{x^3}$ ; $x \neq 0$

find f(0) if $f$ is continuous at $x = 0$

**STEP 1**

$$\lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$$

$$= \lim_{x\to 0} \frac{\frac{\sin x}{\cos x} - \sin x}{x^3}$$

$$= \lim_{x\to 0} \frac{\sin x - \sin x \cdot \cos x}{\cos x \cdot x^3}$$

$$= \lim_{x\to 0} \frac{\sin x\,(1 - \cos x)}{\cos x \cdot x^3}$$

$$= \lim_{x\to 0} \frac{\tan x \cdot 2\sin^2 \frac{x}{2}}{x^3}$$

$$= \lim_{x\to 0} \frac{\tan x}{x} \cdot \frac{2\sin^2 \frac{x}{2}}{x^2}$$

$$= \lim_{x\to 0} \frac{\tan x}{x} \cdot 2\left(\frac{\sin \frac{x}{2}}{x}\right)^2$$

$$= \lim_{x\to 0} \frac{\sin x}{x} \cdot 2\left(\frac{\frac{1}{2}\sin \frac{x}{2}}{\frac{x}{2}}\right)^2$$

$$= \quad 1 \quad . \quad 2\left(\frac{1}{2} \cdot 1\right)^2$$

$$= \quad \frac{1}{2}$$

**STEP 2 :**

Since $f$ is continuous at $x = 0$

$$f(0) = \lim_{x\to 0} f(x)$$

$$= 1/2$$

**Q3. Attempt any TWO of the following**

**(4 marks each) (8 marks )**

**01.** if the f given below is continuous at $x = 2$ and $x = 4$ then find a & b

$$f(x) = \begin{cases} x^2 + ax + b & ; \quad x < 2 \\ 3x + 2 & ; \quad 2 \le x \le 4 \\ 2ax + 5b & ; \quad 4 < x \end{cases}$$

**SOLUTION :**

**PART – 1**

**STEP 1**

$$\lim_{x \to 2-} f(x)$$

$$= \lim_{x \to 2} x^2 + ax + b$$

$$= 2^2 + a(2) + b$$

$$= 4 + 2a + b$$

**STEP 2**

$$\lim_{x \to 2+} f(x)$$

$$= \lim_{x \to 2} 3x + 2$$

$$= 3(2) + 2 = 8$$

**STEP 3**

$$f(2) = 3(2) + 2 = 8$$

**STEP 4**

Since the $f$ is continuous at $x = 2$

$$\lim_{x \to 2-} f(x) = \lim_{x \to 2+} f(x) = f(2)$$

$$4 + 2a + b = 8 = 8$$

$$2a + b = 4 \quad \ldots\ldots\ldots\ldots\ldots\ldots. \quad (1)$$

**PART – 2**

**STEP 1**

$$\lim_{x \to 4-} f(x)$$

$$= \lim_{x \to 4} 3x + 2$$

$$= 3(4) + 2 = 14$$

**STEP 2**

$$\lim_{x \to 4+} f(x)$$

$$= \lim_{x \to 4} 2ax + 5b$$

$$= 2a(4) + 5b$$

$$= 8a + 5b$$

**STEP 3**

$$f(4) = 3(4) + 2 = 14$$

**STEP 4**

Since the $f$ is continuous at $x = 4$

$$\lim_{x \to 4-} f(x) = \lim_{x \to 4+} f(x) = f(4)$$

$$14 = 8a + 5b = 14$$

$$8a + 5b = 14 \quad \ldots\ldots\ldots\ldots\ldots\ldots. \quad (2)$$

Solving (1) and (2) : $a = 3$ , $b = -2$

**02.** find f(0) if $f$ is continuous at x = 0 where

$$f(x) = \frac{x^2}{e^x + e^{-x} - 2} \quad ; \quad x = 0$$

**SOLUTION :**

**Step 1**

$$\lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} \frac{x^2}{e^x + e^{-x} - 2}$$

$$= \lim_{x \to 0} \frac{x^2}{e^x + \frac{1}{e^x} - 2}$$

$$= \lim_{x \to 0} \frac{x^2}{\frac{(e^x)^2 + 1 - 2.e^x}{e^x}}$$

$$= \lim_{x \to 0} \frac{x^2}{\frac{(e^x - 1)^2}{e^x}}$$

$$= \lim_{x \to 0} \frac{e^x}{\frac{(e^x - 1)^2}{x^2}}$$

$$= \lim_{x \to 0} \frac{e^x}{\left(\frac{e^x - 1}{x}\right)^2}$$

$$= \frac{e^0}{(\log e)^2} \qquad = 1$$

**Step 2**

Since $f$ is continuous at x = 0

$$f(0) = \lim_{x \to 0} f(x) = 1$$

**03.** find k if $f$ is continuous at x = 1 where

$$f(x) = \frac{1 + \cos\pi x}{\pi(1-x)^2} \quad ; \quad x \neq 1$$

$$= k \quad ; \quad x = 1$$

**STEP 1**

$$\lim_{x \to 1} f(x)$$

$$= \lim_{x \to 1} \frac{1 + \cos\pi x}{\pi(1-x)^2}$$

**Put x = 1 + h**

$$= \lim_{h \to 0} \frac{1 + \cos\pi(1+h)}{\pi\,[1 - (1+h)]^2}$$

$$= \lim_{h \to 0} \frac{1 + \cos(\pi + \pi h)}{\pi\,[1 - 1 - h]^2}$$

$$= \lim_{h \to 0} \frac{1 - \cos \pi h}{\pi h^2}$$

$$= \lim_{h \to 0} \frac{2 \sin^2 \frac{\pi h}{2}}{\pi h^2}$$

$$= \lim_{h \to 0} \frac{2}{\pi} \frac{\sin^2 \frac{\pi h}{2}}{h^2}$$

$$= \lim_{h \to 0} \frac{2}{\pi} \left(\frac{\sin \frac{\pi h}{2}}{h}\right)^2$$

$$= \lim_{h \to 0} \frac{2}{\pi} \left(\frac{\frac{\pi}{2} \sin \frac{\pi h}{2}}{\frac{\pi h}{2}}\right)^2$$

$$= \frac{2}{\pi} \frac{\pi^2}{4}$$

$$= \frac{\pi}{2}$$

**STEP 2**

Since $f$ is continuous at x = 1

$$f(1) = \lim_{x \to 1} f(x)$$

$$k = \frac{\pi}{2}$$

# Q-4



**Q4. Attempt any THREE of the following**

**(2 marks each) (6 marks )**

**01.** for a bivariate data $b_{yx} = -1.2$ and $b_{xy} = -0.3$ .
Find correlation coefficient between x and y

**SOLUTION**

$r^2 = byx \times bxy$

$r^2 = -1.2 \times -0.3$

$r^2 = \frac{12}{10} \times \frac{3}{10}$

$r^2 = \frac{36}{100}$

$r = \pm \frac{6}{10}$

$r = -\frac{6}{10}$ (byx & bxy are –ve)

**02.** for a bivariate data ;
$\bar{x} = 53$ , $\bar{y} = 28$ , $byx = -1.5$, $bxy = -0.2$ .
Estimate of y for x = 50

**SOLUTION**

$y - \bar{y} = byx(x - \bar{x})$

$y - 28 = -1.5(x - 53)$

$y - 28 = -1.5(50 - 53)$

$y - 28 = -1.5(-3)$

$y - 28 = 4.5$

$y = 32.5$ for $x = 50$

**03.** from the data of 20 pairs of observations on X and Y following results are obtained

$\bar{x} = 199$ ; $\bar{y} = 94$ ; $\Sigma(x - \bar{x})^2 = 1298$

$\Sigma(y - \bar{y})^2 = 600$ ; $\Sigma(x - \bar{x})(y - \bar{y}) = -262$ .

Obtain regression coefficient bxy

**SOLUTION**

$byx = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{\Sigma(x - x)^2}$

$= \frac{-262}{1298}$

$= 0.2018$

| LOG CALC |
|---|
| 2.4183 |
| − 3.1134 |
| AL $\bar{1}.3049$ |
| 0.2018 |

**04.** Regression of two series are $2x - y - 15 = 0$ & $3x - 4y + 25 = 0$ Find mean of x and y

**SOLUTION**

$$
\begin{aligned}
2x - y &= 15 \quad \times 3 \\
3x - 4y &= -25 \quad \times 2 \\
6x - 3y &= 45 \\
6x - 8y &= -50 \\
- \quad + \quad & \quad + \\
5y &= 95 \\
\bar{y} &= 19
\end{aligned}
$$

subs in (1) $\bar{x} = 17$

**Q5. Attempt any TWO of the following** 

**(3 marks each)**

# Q-5

**01.**

| | Sales (crores) (x) | exp.(crores) (y) |
|---|---|---|
| Mean | 40 | 6 |
| S.D | 10 | 1.5 |

correlation coefficient = 0.9

estimate sales for a proposed adv. exp. of ₹ 10 crores

**SOLUTION** **y on x**

$$byx = r \cdot \frac{\sigma y}{\sigma x}$$

$$= 0.9 \times \frac{1.5}{10}$$

$$= \frac{1.35}{10}$$

$$= 0.135$$

$y - \overline{y} = byx(x - \overline{x})$

$y - 6 = 0.135(x - 40)$

$y - 6 = 0.135x - 5.4$

$y = 0.135x - 5.4 + 6$

$y = 0.135x + 0.6$

Put $x = 60$

$y = 0.135(60) + 0.6$

$y = 8.1 + 0.6$

$y = 8.7$ Estimated adv exp. = ₹ 8.7 cr

**03.** Find the equation of line of regression of Y on X for the following data

$n = 8$ ; $\Sigma(x - \overline{x})(y - \overline{y}) = 120$ ; $\overline{x} = 20$ $\overline{y} = 36$ ; $\sigma x = 2$

$\sigma y = 3$

**SOLUTION**

$$\textbf{byx} = \frac{cov(x,y)}{\sigma x^2}$$

$$= \frac{\frac{\Sigma((x - \overline{x})(y - \overline{y})}{n}}{\sigma x^2}$$

$$= \frac{\frac{120}{8}}{4} = \frac{15}{4} = 3.75$$

**Y on X**

$y - \overline{y} = byx\ (x - \overline{x})$

$y - 36 = 3.75(x - 20)$

$y - 36 = 3.75x - 75$

$y = 3.75x - 75 + 36$

$y = 3.75x - 39$

# Q-5

**02.** for 50 students of a class the regression equation of marks in Statistics (x) on the marks in accounts (y) is 3y − 5x + 180 = 0 . The mean of marks of accounts is 44 and variance of marks in Statistics is 9/16th of the variance of marks in accounts . Find mean marks of Statistics and correlation coefficient

**SOLUTION :**

**GIVEN :** X ON Y : $3y - 5x + 180 = 0$

$$\overline{y} = 44$$

$$\frac{\sigma x^2}{\sigma y^2} = \frac{9}{16}$$

**STEP 1**

X ON Y : $3y - 5x + 180 = 0$

$$5x = 3y + 180$$

$$x = \frac{3y}{5} + \frac{180}{5}$$

$$bxy = \frac{3}{5}$$

**STEP 2**

$$bxy = r \cdot \frac{\sigma x}{\sigma y}$$

$$\frac{3}{5} = r \times \frac{3}{4}$$

$$r = \frac{4}{5}$$

**STEP 3**

Put y = 44 in

$$x = \frac{3y}{5} + \frac{180}{5}$$

$$x = \frac{3(44) + 180}{5}$$

$$x = \frac{132 + 180}{5}$$

$$x = \frac{312}{5}$$

$$x = 62.4$$

mean marks in statistics = 62.4

**Q6. Attempt any TWO of the following**

**(4 marks each)**

# Q-6

**01.** x : index of production
y : no. of unemployed (in lacs)

find regression line y on x

| x | y | $x-\bar{x}$ | $y-\bar{y}$ | $(x-\bar{x})^2$ | $(y-\bar{y})^2$ | $(x-\bar{x})(y-\bar{y})$ |
|---|---|---|---|---|---|---|
| 100 | 15 | −4 | 0 | 16 | | 0 |
| 102 | 12 | −2 | −3 | 4 | | 6 |
| 104 | 13 | 0 | −2 | 0 | | −0 |
| 107 | 11 | 3 | −4 | 9 | | −12 |
| 105 | 12 | 1 | −3 | 1 | | −3 |
| 112 | 12 | 8 | −3 | 64 | | −24 |
| 103 | 19 | −1 | 4 | 1 | | −4 |
| 99 | 26 | −5 | 11 | 25 | | −55 |
| 832 | 120 | 0 | 0 | 120 | | −98 + 6 = −92 |
| $\Sigma x$ <br> $\bar{x} = 104$ | $\Sigma y$ <br> $\bar{y} = 15$ | | | $\Sigma(x-\bar{x})^2$ | $\Sigma(y-\bar{y})^2$ | $\Sigma(x-\bar{x})(y-\bar{y})$ |

$$byx = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-x)^2}$$

$$= \frac{-92}{120}$$

$$= -0.77$$

$$y - \bar{y} = byx\,(x - \bar{x})$$

$$y - 15 = -0.77(x - 104)$$

$$y - 15 = -0.77x + 80.08$$

$$y = -0.77x + 80.08 + 15$$

$$y = -0.77x + 95.08$$

**02.**

Information on vehicles (in thousands) passing through seven different highways during a day (X) and number of accidents reported (Y) is given as

$\Sigma x = 105$ ; $\Sigma y = 409$ ; $\Sigma x^2 = 1681$ ; $\Sigma y^2 = 39350$ $\Sigma xy = 8075$

Obtain linear regression of Y on X

**SOLUTION**

$$\bar{x} = \frac{\Sigma x}{n} = \frac{105}{7} = 15 \qquad \bar{y} = \frac{\Sigma y}{n} = \frac{409}{7} = 58.43$$

$$byx = \frac{n\Sigma xy - \Sigma x.\Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{7(8075) - (105)(409)}{7(1681) - (105)^2}$$

$$= \frac{56525 - 42945}{11767 - 11025}$$

$$= \frac{13580}{742}$$

$$= 18.30$$

LOG CALC

4.1329
− 2.8704

AL 1.2625

18.30

Equation

$$y - \bar{y} = byx\,(x - \bar{x})$$

$$y - 58.43 = 18.30(x - 15)$$

$$y - 58.43 = 18.30x - 274.5$$

$$y = 18.30x - 274.50 + 58.43$$

$$y = 18.30x - 216.07$$

**03.** the equations of two regression lines are

$2x + 3y - 6 = 0$ & $5x + 7y - 12 = 0$
Find correlation coefficient

**STEP 1**

ASSUME

XON Y : $5x + 7y - 12 = 0$

$5x = -7y + 12$

$x = \frac{-7y}{5} + \frac{12}{5}$

$bxy = -\frac{7}{5}$

Y ON X : $2x + 3y - 6 = 0$

$3y = -2x + 6$

$y = -\frac{2}{3} y + \frac{6}{3}$

$byx = \frac{-2}{3}$

**STEP 2**

$r^2 = bxy \, . \, byx$

$= \frac{-7}{4} \times \frac{-2}{3}$

$= \frac{14}{15}$

Since $0 \le r^2 \le 1$
Our assumptions are correct

$r = \pm\sqrt{\frac{14}{15}}$

$r = -\sqrt{\frac{14}{15}}$ ( byx & bxy are –ve)

$\log r' = \frac{1}{2}\left(\log 14 - \log 15\right)$

$\log r' = \frac{1}{2}\left(1.1461 - 1.1761\right)$

$\log r' = \frac{1.1461}{2} - \frac{1.1761}{2}$

$\log r' = 0.5730 - 0.5880$

$\log r' = \bar{1} \, . \, 9850$

$r' = AL(\bar{1} \, . \, 9850)$

$r' = 0.9661$ $r = -0.9661$